

Dilemma game in a traffic model with the crossing*

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In this paper, we investigate the non-signalized intersection issue considering traffic flow and energy dissipation in terms of game theory based on the Nagel-Schreckenberg (NaSch) model. There are two types of driver agents at the intersection when vehicles on the two streets are approaching to it simultaneously: C agents (cooperative strategy) pulling up to avoid collision and D agents (defective strategy) crossing the intersection audaciously. Phase diagram of the system, which describes free-flow phase, segregation phase, jammed phase and maximum current curve representing the social maximum payoff, is presented. Dilemma game is observed at the phase-segregated state except for the case of $v_{\max} = 1$.

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I. INTRODUCTION

Recently, traffic problems have attracted much attention of a community of physicists because of the observed nonequilibrium phase transitions and various non-linear dynamical phenomena. In order to investigate the dynamical behavior of the traffic flow, a number of traffic models such as fluid dynamical models, gas-kinetic models, car-following models and cellular automata (CA) models[1–4] have been proposed. These dynamical approaches represented complex physical phenomena of traffic flow among which are hysteresis, synchronization, wide moving jams, and phase transitions, etc. Among these models, the advantages of CA approaches, which have been extensively applied and investigated, show the flexibility to adapt complicated features observed in real vehicular traffic[1, 4, 5]. The Nagel-Schreckenberg (NaSch) model is a basic CA models describing one-lane traffic flow[6]. Based on the NaSch model, many CA models have been extended to investigate the properties of the system with realistic traffic factors such as highway junctions, crossing, tollbooths and speed limit zone[1, 4, 7–10].

Previously, scholars pay more attention to traffic flow while investigating vehicular traffic issues. Most recently, the problems of energy dissipation in traffic system have been investigated widely[11–17] for environmental pollution and energy dissipation caused by vehicular traffic have become more and more significant in modern society. Intersections are fundamental units of complex city traffic networks. Optimization of traffic flow and energy consumption at a isolated intersection is a substantial ingredient for the task of global optimization of city networks. During the past ten years, physicists

have paid notable attention to controlling traffic flow at intersections[8, 18–22]. However, to our knowledge, none of these previous studies about intersections issue present energy dissipation information, which should be further investigated.

Signal control works only for major intersections but in most cases, signal system is not installed due to cost. Drivers at the intersection without signal system can only communicate each other by eye contact and make a decision based on own judgment. Most of the previous studies focus only on the kinetics of the self-driven multi particle system and ignore the effect of drivers' decisions on the entire system. In this paper, considering traffic flow and energy dissipation, we add a game theory framework[23–25] as a rational decision process to the traffic model with a non-signalized intersection, and demonstrate that the intersection has a dilemma structure. In addition, the phase diagram which shows the social maximum payoff is presented.

The paper is organized as follows. Section II is devoted to the description of the problem. In section III, the results of the numerical experiment are given and discussed. Finally, the conclusions are given in section IV.

II. DESCRIPTION OF THE PROBLEM

In this section, we present a CA model with two perpendicular one dimensional closed chains. The chains represent urban streets accommodating unidirectional vehicular traffic flow. The direction of traffic flow in the first chain is from south to north and from east to west in the second chain, as shown in figure 1. Each street consists of L cells of equal size numbered by $i = 1, 2, \dots, L$ and the time is discrete. The two chains intersect each other at the sites $i_1 = i_2 = L/2$ on the first and second chains respectively. Each site can be either empty or occupied by a vehicle with the speed $v = 0, 1, 2, \dots, v_{\max}$, where v_{\max} is the speed limit. Let $x(i, t)$ and $v(i, t)$ de-

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note the position and the velocity of the i th car at time t , respectively. The number of empty cells in front of the i th vehicle is denoted by $d(i, t) = x(i+1, t) - x(i, t) - 1$. The evolution dynamics is based on the Nagel-Schreckenberg (NaSch) model. The updating rules of the NaSch model are as follows:

(1) Acceleration:

$$v(i, t + 1/3) \rightarrow \min[v(i, t) + 1, v_{\max}];$$

(2) Slowing down:

$$v(i, t + 2/3) \rightarrow \min[v(i, t + 1/3), d(i, t)];$$

(3) Stochastic braking:

$$v(i, t+1) \rightarrow \max[v(i, t+2/3)-1, 0] \text{ with the probability } p;$$

(4) Movement: $x(i, t + 1) \rightarrow x(i, t) + v(i, t + 1)$.

The above four steps for all vehicles update in parallel with periodic boundary.

Vehicles without interactions of vehicles on the perpendicular streets evolve under the NaSch dynamics. However, how does the vehicle approaching to the intersection evolve when vehicle on the other street approaches to the intersection simultaneously? The approaching driver to the intersection need considering not only the condition of it's front vehicle but also the situation of the approaching vehicle on the perpendicular street. Different drivers perform differently even at the same condition. The decision-making process of the driver approaching to the intersection is described by game theory, i.e., we assume that drivers have a strategy that is either co-operative or defective. Cooperative drivers (C agents) pull up in the front of the intersection to avoid collision. Defective drivers (D agents) cross the intersection audaciously. At the same time, if the two drivers approaching to the intersection on the two streets are all D agents, i.e., the two drivers adopt defective strategy simultaneously, traffic accident would occur. Different from "the prisoners' dilemma" [26], one may find that "non-tit-for-tat" (I'll cooperate (defect) with you if you defect (cooperate) with me) is a comparatively effective strategy for playing the drivers's dilemma. During the simulation, to avoid collision, we assume that at the same time if the driver approaching to the intersection on the first street is D agent (C agent), the driver approaching to the intersection on the second street adopts cooperative strategy (defective strategy). The probability of the situation that two approaching drivers to the intersection are D agents or C agents to occur is very small. In most cases of real traffic, only one approaching driver to the intersection is D agent and the other is C agent. Let P_d denotes the probability that the driver approaching to the intersection on the first street adopt defective strategy when the other driver on the second street is also approaching to the intersection at the same time.

The payoff indicates traffic flow J which is the product of the mean velocity and vehicle density. Except for traffic flow, energy problem is an important issue in traffic system. The kinetic energy of the vehicle with the velocity v is $mv^2/2$, where m is the mass of the vehicle. When braking the kinetic energy reduces. Let E_d

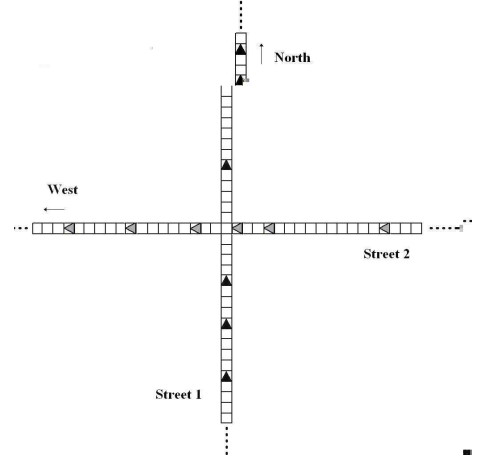


FIG. 1: Illustration of the intersection of two uni-directional streets with periodic boundary. They cross each other at halfway.

denotes energy dissipation rate per time step per vehicle. For simple, we neglect rolling and air drag dissipation and other dissipation such as the energy needed to keep the motor running while the vehicle is standing in our analysis, i.e., we only consider the energy lost caused by speed-down. The dissipated energy of i th vehicle from time $t - 1$ to t is defined by

$$e(i, t) = \begin{cases} \frac{m}{2} [v^2(i, t-1) - v^2(i, t)] & \text{for } v(i, t) < v(i, t-1) \\ 0 & \text{for } v(i, t) \geq v(i, t-1). \end{cases} \quad (1)$$

Thus, the energy dissipation rate

$$E_d = \frac{1}{T} \frac{1}{N} \sum_{t=t_0+1}^{t_0+T} \sum_{i=1}^N e(i, t), \quad (2)$$

where N is the number of vehicles in the system and t_0 is the relaxation time, taken as $t_0 = 1.5 \times 10^4$. In this model, the particles are "self-driven" and the kinetic energy increases in the acceleration step. In the stationary state, the value of the increased energy while accelerating is equivalent to that of the dissipated energy caused by speed-down, and the kinetic energy is constant in the system.

In the simulation, the system size $L = 500$ and $N_1 = N_2$ are selected where N_1 (N_2) is the number of vehicles on the first street (second street), and the stochastic braking is not considered, i.e., $p = 0$. The numerical results are obtained by averaging over 20 initial configurations and 5×10^3 time steps after discarding 1.5×10^4 initial transient states.

III. NUMERICAL RESULTS

First of all, we investigate the influences of the drivers' decision on the social average payoff based on the de-

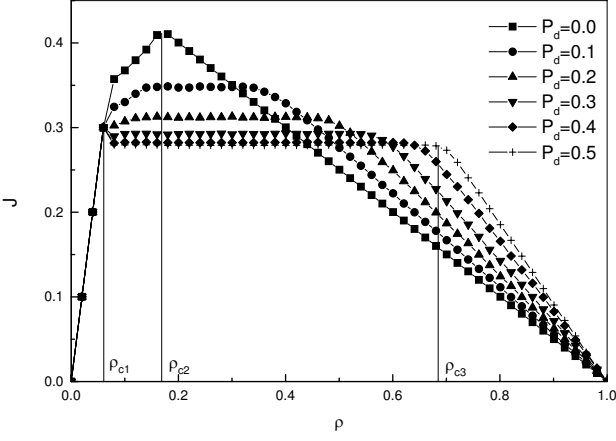


FIG. 2: The social average payoff J as a function of the vehicle density ρ in the case of $v_{\max} = 5$ and $p = 0$ for various values of the probability P_d . The social average payoff indicates the mean current of the traffic system with a single intersection.

terministic NaSch model with the speed limit $v_{\max} = 5$. Figure 2 shows the average payoff J as a function of the vehicle density ρ with the probability $0 \leq P_d \leq 0.5$. Because of the equivalence of the two streets, the condition inverts while $0.5 \leq P_d \leq 1$. As shown in Fig. 2, there is a critical density $\rho_{c1} = 0.0625$ below which P_d has no influence on social average payoff and J increases linearly with the vehicle density. Above the critical density ρ_{c1} , J undergoes a short rapid increase or decrease after which the plateau arises whose height and length are determined by the probability P_d . After the plateau, J exhibits linear decrease with the increase of vehicle density. The intersection of the two streets makes the crossing point appear as a sideways dynamical defective site. The localized defect has global effects whereby the traffic exhibits macroscopic phase segregation into low-density and high-density regions. For the first street, the smaller the probability P_d is, the stronger is the dynamic defect. Considering the payoff of each street, the larger the probability that an approaching driver to the intersection adopts defective strategy, the larger the payoff of the driver and the street on which the agent drivers are. However, for different vehicle density, the maximum social mean payoff corresponds to different values of P_d .

Except for ρ_{c1} , there are two critical density $\rho_{c2} = 0.167$ below which J is largest in the case of $P_d = 0.0$ and $\rho_{c3} = 0.67$ above which J is largest in the case of $P_d = 0.5$. For the whole density region, the maximal social average payoff $J_{\max} = v_{\max}/2(v_{\max} + 1)$ appears at the critical density ρ_{c2} .

Figure 3 exhibits the relation of J to the probability P_d with various values of the vehicle density ρ in the case of $v_{\max} = 5$. As expected, the symmetry center of the curve is at $P_d = 0.5$ for the two streets are equivalent in our model. While $\rho > \rho_{c3}$, with the increase of the probability P_d , J first increases and then decreases after a maximum value is reached. In the density interval

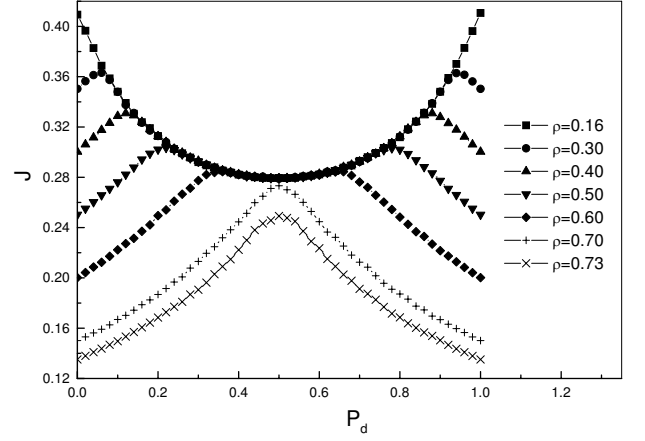


FIG. 3: The social average payoff J as a function of the probability P_d in the case of $v_{\max} = 5$ and $p = 0$ for various values of the vehicle density ρ .

$\rho_{c2} < \rho < \rho_{c3}$, J increases with P_d to the maximum, then it decreases with P_d until $P_d = 0.5$. After the point, J exhibits an increase and decreases subsequently after the maximum is reached. In the density interval $\rho_{c1} < \rho < \rho_{c2}$, with the increases of the probability P_d , J first decreases and increases after a minimum value is reached. It is noted that when $\rho_{c1} < \rho < \rho_{c2}$ at the probability P_d interval that the two maximal J appears, curves collapse into one curve which is the social payoff at ρ_{c1} .

Except for fundamental diagram (J versus ρ), it is worthwhile to investigate the energy dissipation diagram of the system with a intersection. Figure 4 shows the relation of energy dissipation rate E_d to the vehicle density ρ with various values of P_d in the case of $v_{\max} = 5$. As shown in Fig. 4, there are three critical density ρ_{c1} below which no energy dissipation occurs, ρ_{c2} at which there is no energy dissipation in the case of $P_d = 0.0$, and ρ_{c3} above which E_d is largest when $P_d = 0.5$. While $\rho \rightarrow \rho_{c1}$, with the decrease of P_d , energy dissipation rate E_d reduces. And while $P_d = 0.0$, E_d is minimal in the density interval $\rho_{c1} < \rho < \rho_{c2}$, which is contrary to traffic flow J . The value of E_d decreases as P_d increases in the middle density region, but increases in the high density region.

From the viewpoint of individual benefit, adopting the higher payoff strategy is more rational than using the opposite strategy. For agents on the first street, the larger the P_d is, the more payoff they obtain. For agents on the second street, the smaller the P_d is, the more payoff they acquire. When the system reaches equilibrium state, the probability that drivers approaching to the intersection adopt defective strategy is 0.5. However, when $P_d = 0.5$ the average social payoff is not maximum, but minimum in the density interval $\rho_{c1} < \rho < \rho_{c3}$, which is a social dilemma.

From Fig. 2 and 4, one should noted that in the density interval $\rho_{c1} < \rho < \rho_{c2}$, if $P_d = 0.0$ i.e., drivers on the first street (second street) are all C agents (D agents), the

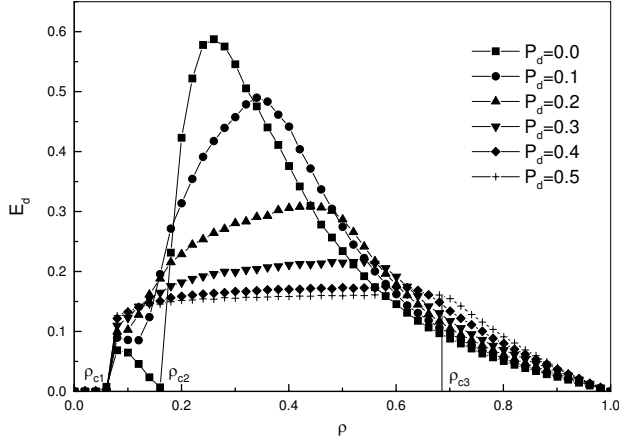


FIG. 4: Energy dissipation rate E_d (scaled by m) as a function of the vehicle density ρ in the case of $v_{\max} = 5$ and $p = 0$ for various values of the probability P_d .

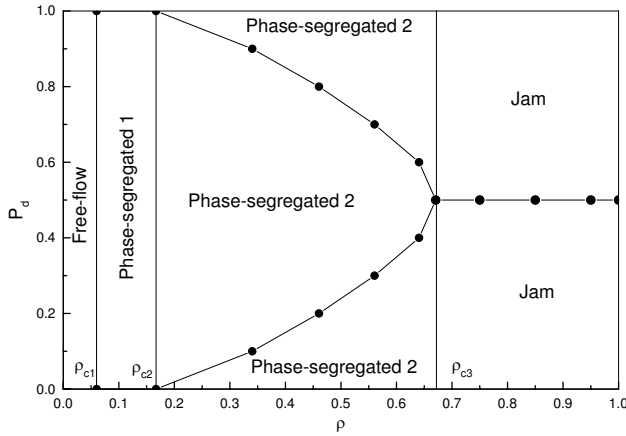


FIG. 5: Illustration of the ρ - P_d phase diagram in the case of $v_{\max} = 5$ and $p = 0$. The closed circles in the diagram represent the maximum payoff for different vehicle density.

payoff of the whole system is maximal and energy dissipation is minimal. The best situation having high social efficiency is that drivers on one of the two streets always pull up and let drivers on the other street cross, while the interactions of vehicles on the two streets emerges. However, C agents pulling up for a long time has a robust incentive to adopt defective strategy for D agents can obtain higher payoff than C agents. Thus, the probability P_d always increases, finally reaching absorbed equilibrium $P_d = 0.5$, where the probability that D agents appear on the two streets is the same.

When $\rho > \rho_{c3}$, the internal equilibrium point $P_d = 0.5$ is consistent with the social maximum payoff where no social dilemma occurs. This implies that in the jam state D agents appearing on the two streets with the same probability can improve flow efficiency of the system, rather than only drivers on the second street adopt defective strategy.

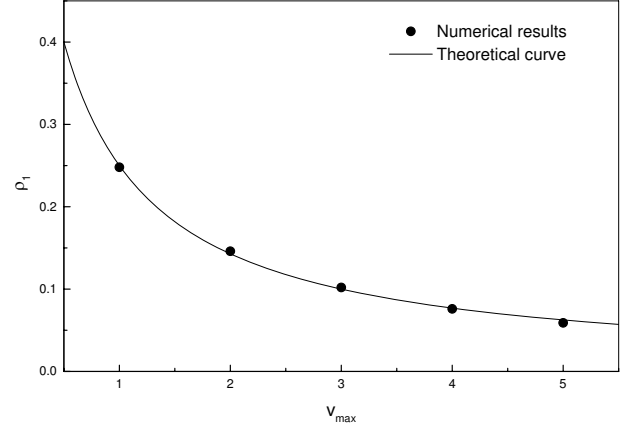


FIG. 6: The critical density ρ_{c1} as a function of the speed limit v_{\max} in the case of $p = 0$. Symbol data are obtained from computer simulations, and solid line corresponds to analytic results of the formula (3).

Figure 5 shows the ρ - P_d phase diagram with the speed limit $v_{\max} = 5$. There are four traffic phases: free-flow, phase-segregated 1, phase-segregated 2 and jammed phase, which are separated by the critical density ρ_{c1} , ρ_{c2} and ρ_{c3} , as shown in Fig. 5. The solid circle symbols in Fig.5 represent the maximum social payoff. In the free-flow phase in which vehicles can move freely, there are no interactions of vehicles in the system and no energy dissipation to occur. In the phase-segregated region, the macroscopic traffic phase segregates into high-density and low-density region. In the phase-segregated 1 state, the maximal current consists with the minimal energy dissipation rate. The maximum social payoff appears at $P_d = 0.0$ or $P_d = 1.0$. In the phase-segregated 2 state, however, the maximal current consists with the maximal energy dissipation rate and E_d increases with traffic flow J . The probability P_d consistent with the maximal current increases exponentially with the increase of vehicle density while $P_d < 0.5$. However, when $0.5 < P_d < 1.0$, P_d consistent with the maximal current decreases with the increase of ρ . In the jammed phase, the probability P_d consistent with the maximal current is independent of ρ and equals 0.5.

Next, we quantitatively analyze the critical density ρ_{c1} , ρ_{c2} and ρ_{c3} for different speed limit v_{\max} . While the mean distance-headway is greater than $3v_{\max} + 1$, there are no interactions of vehicles in the system and vehicles on the perpendicular streets can move freely. Consequently, the critical density ρ_{c1} below which vehicles move freely and no energy dissipation occurs, can be written as

$$\rho_{c1} = \frac{1}{3v_{\max} + 1}. \quad (3)$$

Figure 6 exhibits the relation of the critical density ρ_{c1} to the speed limit v_{\max} . Formula (3) gives an agreement with numerical results in Fig.6. At the critical den-

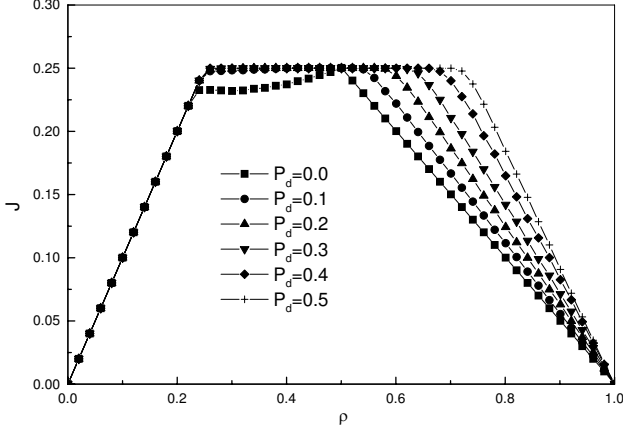


FIG. 7: The social average payoff J as a function of the vehicle density ρ in the case of $v_{\max} = 1$ and $p = 0$ for various values of the probability P_d .

sity ρ_{c2} , vehicles on the second street (first street) can move freely in the case of $P_d = 0.0$ ($P_d = 1.0$). Thus, the critical density ρ_{c2} is given as

$$\rho_{c2} = \frac{1}{v_{\max} + 1}. \quad (4)$$

Above the critical density ρ_{c3} , high-density region expands into the whole system. The critical point ρ_{c3} is not determined by the speed limit v_{\max} and in the case of $P_d = 0.5$, ρ_{c3} can be written as

$$\rho_{c3} = \frac{1/P_d}{1 + 1/P_d} = \frac{3}{2}. \quad (5)$$

However, for the case of $v_{\max} = 1$, traffic flow and energy dissipation exhibit different features. As shown in Fig. 7, there is only one plateau whose height value is equal to 0.25 for different values of P_d . The length of the plateau increases with the increase of P_d , for $0.0 < P_d \leq 0.5$. In the case of $P_d = 0.0$, there is no plateau and the maximal J , whose value equals to 0.25, appears at the critical density point ρ_{c2} , above which J exhibits linear decrease with the increase of vehicle density. After the critical density ρ_{c1} , the social average payoff J is maximum in the case of $P_d = 0.5$ and is minimum in the case of $P_d = 0.0$. Consequently, there is no social dilemma in the case of $v_{\max} = 1$.

In the plateau region, for the case of $v_{\max} = 1$, the probability that a approaching vehicle on the first street crosses the intersection per time step is $P_d/2$ and is $(1-P_d)/2$ on the second street. Thus, the social average payoff in the plateau region is 0.25 and does not depend on P_d .

Figure 8 shows energy dissipation rate E_d as a function of the vehicle density ρ with various values of the probability P_d in the case of $v_{\max} = 1$. As shown in Fig.8,

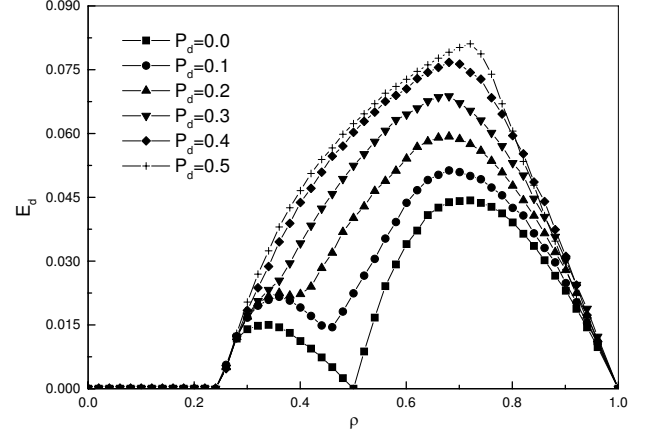


FIG. 8: Energy dissipation rate E_d (scaled by m) as a function of the vehicle density ρ in the case of $v_{\max} = 1$ and $p = 0$ for various values of the probability P_d .

there are two critical density ρ_{c1} below which no energy dissipation occurs, and ρ_{c2} at which no go-and-stop vehicles appear when $P_d = 0.0$. The energy dissipation rate E_d increases with the increase of P_d for $0.0 \leq P_d \leq 0.5$, which is different from that for $v_{\max} > 1$. Considering the ρ - P_d phase diagram, there is no differences between phase-segregated 1 and phase-segregated 2, and P_d consistent with the maximal social payoff is always equal to 0.5 and independent of ρ in phase-segregated and jammed states (not shown). Therefore these results indicate that different correlations of spacetime exist between the case of $v_{\max} = 1$ and $v_{\max} > 1$.

IV. SUMMARY

In this paper, we investigated the social dilemma structure in a traffic model with a non-signalized intersection based on the NaSch model. The model contains a game theory framework to deal with drivers' decision-making processes. We studied the effects of the drivers' decision on traffic flow and energy dissipation at different traffic phases.

Numerical results indicate that in the case of $v_{\max} > 1$ the social dilemma appears at the phase-segregated states and no dilemma exists at other traffic phases. At the phase-segregated states, selfish drivers crossing the intersection can obtain a higher payoff than altruistic drivers pulling up in the front of the intersection, but they cause a remarkable decrease in social efficiency when they emerge alternately on the two streets. In contrast to the phase-segregated states, in the jammed phase, the social efficiency is maximal at the absorbed equilibrium $P_d = 0.5$. Different from that in the case of $v_{\max} > 1$, in the case of $v_{\max} = 1$, there is no dilemma to occur no matter in the phase-segregated and jammed states.

In addition, the three critical density ρ_{c1} , ρ_{c2} and

ρ_{c3} were analyzed quantitatively and theoretical analyses give an excellent agreement with numerical results. However, explicit expressions about the maximum so-

cial payoff curve in the ρ - P_d phase diagram do not be obtained because of effects of long length of time space correlations, and deserve further investigate.

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